Monotonicity-based Electrical Impedance Tomography Lung Imaging

Liangdong Zhou*, Bastian Harrach†, and Jin Keun Seo* Member, IEEE.
*Department of Computational Science & Engineering, Yonsei University, Korea
†Department of Mathematics, University of Stuttgart, Germany.

Abstract—This paper presents a monotonicity-based spatiotemporal conductivity imaging method for continuous regional lung monitoring using electrical impedance tomography (EIT). The EIT data (i.e., the boundary current-voltage data) can be decomposed into pulmonary, cardiac and other parts using their different periodic natures. The time-differential current-voltage operator corresponding to lung ventilation can be viewed as either semi-positive or semi-negative definite owing to monotonic conductivity changes within the lung region. We used this monotonicity constraint to improve the quality of lung EIT imaging. We tested the proposed method via numerical simulations and phantom experiments.

Index Terms—Electrical impedance tomography, continuous lung monitoring, monotonicity, inverse problem, monotonicity-based regularization

I. INTRODUCTION

ELECTRICAL impedance tomography (EIT) received much recent attention owing to its unique ability to allow long-term, continuous monitoring of lung ventilation at the bedside [2], [3], which is not possible using other medical imaging techniques such as computerized tomography, magnetic resonance imaging, ultrasound, or SPECT. Increasing demands for monitoring intensive-care patients have recently led commercial EIT systems to be considered as a means to guide strategies of protective lung ventilation through their use to monitor closely the patient’s lung condition [7], [13], [14], [28], [30], [33], [39]. Although EIT cannot compete with CT, MRI or ultrasound in terms of spatial resolution or accuracy [24], its ability to provide long-term, continuous monitoring and portability make it clinically useful.

Numerous efforts have sought to develop robust EIT reconstruction algorithms since the invention of the first EIT devices by Barber and Brown in the early 1980s [1], [5], [6], [32]. However, EIT is not yet suitable for routine clinical use because of its poor sensitivity [36]. Robust reconstruction may rely on incorporating some strong prior information into the algorithm via regularization.

This paper focuses mainly on lung EIT. In EIT, multiple surface electrodes are attached to an imaging object to inject currents and measure boundary voltages as shown in Fig. 1. At low frequencies below 100 KHz, the potential induced by the injection current is dictated approximately by the elliptic partial differential equation \( \nabla \cdot (\sigma \nabla u) = 0 \) inside the object where \( \sigma \) is the time-varying conductivity distribution associated with lung ventilation [24], [35]. The measured EIT data can be viewed as a boundary current-voltage map from the injection current to the resulting boundary voltage, which is determined mainly by the effective conductivity distribution, the configuration of the surface electrodes, and the geometry of the imaging object. Lung EIT aims to provide dynamic images of the time-differential conductivity distribution from the time-differential current-voltage map while minimizing the forward modeling errors due to such as electrode position and boundary geometry and uncertainty of the reference conductivity distribution.

![Fig. 1. Electrodes configuration on chest phantom.](image)

Given the fundamental limitation of EIT, strong prior information about the individual’s pulmonary function and the measured data need to incorporated into the analysis [38]. We know that time-varying patterns in EIT data depend mainly on the breathing and cardiac cycles and diaphragm motion. Since there are very different frequencies of the breathing and cardiac cycles, we can extract patterns corresponding to respiratory motion from the EIT data by eliminating the signals associated with heart motion and other process [15], [29], [34].

Using the extracted ventilation rate signals, we apply the characteristic of monotonicity to the regional lung imaging. This is based on the assumption that changes of conductivity distribution in the lung region are either monotonically non-increasing or monotonically non-decreasing at any fixed time. An increase of conductivity decreases the voltage measurements in terms of matrix definiteness. Conversely, a conductivity decrease would increase the voltage measurements. We observed that the time-derivative of the current-voltage data associated with ventilation is a positive operator during inhalation and a negative operator during exhalation.

We therefore enforce a global non-positivity constraint on the reconstructed conductivity changes during inhalation, and a...
global non-negativity constraint during exhalation. Moreover, on each pixel, we derive a local lower bound during inhalation (and a local upper bound during exhalation) using a sensitivity-based variant of the linearized monotonicity method for inclusion detection [22]. Enforcing the monotonicity constraints in the image reconstruction algorithm can compensate for the inherent ill-conditioned nature of EIT [4]. The effectiveness of the proposed methods were validated by numerical simulations and then demonstrated in phantom experiments.

II. METHODS

A. Mathematical model

Let an imaging object occupy a two- or three-dimensional region $\Omega \subset \mathbb{R}^n$ $(n = 2, 3)$ bounded by its surface $\partial \Omega$. We denote the time-varying conductivity at a position $x$ and time $t$ as $\sigma^t(x)$. In $E$-channel EIT system as shown in Fig. 2, we attach surface electrodes $E_j$ for $j = 1, 2, \ldots, E$ on the boundary $\partial \Omega$ to inject $E$ different currents $(E - 1$ linearly independent) using orderly chosen pairs of electrodes. For the ease of explanation, we assume that $j$-th injection current is made by the adjacent pair of $E_{k}$ and $E_{k+1}$, and all currents are dc. Here, and in the following, we use the convention that $E_{k+1} = E_1$. Then, the distribution of the voltage subject to the $j$-th injection current, denoted by $u_j^t$, is governed by the following equations (the so-called shunt electrode model, cf. [10])

$$\nabla \cdot (\sigma^t(x) \nabla u_j^t(x)) = 0 \quad \text{for } x \in \Omega, \quad (1)$$

$$\sigma^t(x) \nabla u_j^t(x) \cdot n(x) = 0 \quad \text{for } x \in \partial \Omega \setminus \bigcup_{k=1}^{E} E_k, \quad (2)$$

$$u_j^t(x)|_{E_k} = \text{const.} \quad \text{on each } E_k, \ k = 1, \ldots, E, \quad (3)$$

and, for $k = 1, \ldots, E$,

$$\int_{E_k} \sigma^t(x) \nabla u_j^t(x) \cdot n(x) \, ds = \begin{cases} I & \text{for } k = j, \\ -I & \text{for } k = j + 1, \\ 0 & \text{else,} \end{cases} \quad (4)$$

where $n$ is the outward unit normal vector on $\partial \Omega$ and contact impedances are ignored for simplicity. The magnitude of the current driven through the $j$-th and the $(j + 1)$-th electrode is assumed to be normalized to $I = 1$. The solution $u_j^t$ is unique up to constant factors.

Assuming that boundary voltages between all adjacent pairs of electrodes are measured, the $k$-th boundary voltage measured between $E_k$ and $E_{k+1}$ subject to the $j$-th injection current is the time-varying function

$$V_{j,k}^t = u_j^t|_{E_k} - u_j^t|_{E_{k+1}}, \quad j, k = 1, 2, \ldots, E. \quad (5)$$

Thus, we collect $E^2$ number of time-varying boundary data which can be expressed as the following matrix form:

$$V(t) = \begin{bmatrix} V_{1,1}^1(t) & \cdots & V_{1,E}^1(t) \\ V_{2,1}^1(t) & \cdots & V_{2,E}^1(t) \\ \vdots & \ddots & \vdots \\ V_{E,1}^1(t) & \cdots & V_{E,E}^1(t) \\ V_{1,1}^2(t) & \cdots & V_{1,E}^2(t) \\ V_{2,1}^2(t) & \cdots & V_{2,E}^2(t) \\ \vdots & \ddots & \vdots \\ V_{E,1}^2(t) & \cdots & V_{E,E}^2(t) \\ \vdots & \ddots & \vdots \\ V_{1,1}^E(t) & \cdots & V_{1,E}^E(t) \\ V_{2,1}^E(t) & \cdots & V_{2,E}^E(t) \\ \vdots & \ddots & \vdots \\ V_{E,1}^E(t) & \cdots & V_{E,E}^E(t) \end{bmatrix}. \quad (6)$$

The inverse problem of lung EIT for monitoring lung function is to visualize the time varying distribution of $\sigma^t$ in the lung region from the time-varying data $V(t)$.

B. Data separation and monotonicity

The conductivity is related to the measured signals by the identities

$$V_{j,k}^t = \int_{\Omega} \sigma^t \nabla u_j^t \cdot \nabla u_k^t \, dx, \quad (7)$$

$$\frac{d}{dt} V_{j,k}^t = -\int_{\Omega} \frac{\partial \sigma^t}{\partial t} \nabla u_j^t \cdot \nabla u_k^t \, dx, \quad (8)$$

which are proven in the Appendix A. The potentials $u_j^t$ and $u_k^t$ depend on the unknown conductivity $\sigma^t$. Standard linearized reconstruction methods [35] for lung EIT replace $u_j^t$ and $u_k^t$ on the right hand side of (7) by reference potentials and solve the resulting linear equation to determine $\frac{\partial \sigma^t}{\partial t}$ from $\frac{d}{dt} V_{j,k}^t$.

In this work, we will propose two monotonicity-based improvements to this classical linearized reconstruction method. The first improvement is based on the following data separation approach. We decompose the time-derivative of the conductivity $\frac{\partial \sigma^t}{\partial t}$ at position $x$ into two parts:

$$\frac{\partial \sigma^t}{\partial t}(x) = \frac{\partial \sigma^t_L}{\partial t}(x) + \frac{\partial \sigma^t_H}{\partial t}(x) \quad x \in \Omega, \quad (9)$$

where $\frac{\partial \sigma^t_L}{\partial t}$ and $\frac{\partial \sigma^t_H}{\partial t}$ are generated by lung motion and the other motions including heart motion, respectively.

For the application of lung monitoring, we are interested in recovering the lung motion $\frac{\partial \sigma^t_L}{\partial t}$. Hence, we postulate the existence of a respiratory motion-related signal

$$V_{L,j,k}^t = \int_{\Omega} \sigma^t_L \nabla u_j^t \cdot \nabla u_k^t \, dx, \quad (10)$$

and aim to extract

$$\frac{d}{dt} V_{L,j,k}^t = -\int_{\Omega} \frac{\partial \sigma^t_L}{\partial t} \nabla u_j^t \cdot \nabla u_k^t \, dx, \quad (11)$$

from $\frac{d}{dt} V_{j,k}^t$. As above, the lung-related data is written as an $E \times E$-matrix

$$V_L(t) = \begin{bmatrix} V_{L,1,1}^1(t) & \cdots & V_{L,1,E}^1(t) \\ V_{L,2,1}^1(t) & \cdots & V_{L,2,E}^1(t) \\ \vdots & \ddots & \vdots \\ V_{L,E,1}^1(t) & \cdots & V_{L,E,E}^1(t) \\ V_{L,1,1}^2(t) & \cdots & V_{L,1,E}^2(t) \\ V_{L,2,1}^2(t) & \cdots & V_{L,2,E}^2(t) \\ \vdots & \ddots & \vdots \\ V_{L,E,1}^2(t) & \cdots & V_{L,E,E}^2(t) \\ \vdots & \ddots & \vdots \\ V_{L,1,1}^E(t) & \cdots & V_{L,1,E}^E(t) \\ V_{L,2,1}^E(t) & \cdots & V_{L,2,E}^E(t) \\ \vdots & \ddots & \vdots \\ V_{L,E,1}^E(t) & \cdots & V_{L,E,E}^E(t) \end{bmatrix}. \quad (12)$$

The respiratory motion-related signal $V_{L,j,k}^t$ can be obtained by applying band-pass filter to the measured boundary data with a selected frequency range, because the frequency spectrum of the respiratory motion is different from that of cardiac motion. We illustrate the numerical simulation of boundary data decomposition and monotonicity relation of decomposed lung signal and conductivity variation in Fig. 2 where $V_{L,j,k}^t$ stands for the boundary data related to the cardiac motion and other effects. We can see that the separated signals (d) in Fig. 2 is exactly of the same pattern with (b).

After extracting the lung motion part $V_{L,j,k}^t$ from the measured signals using a band-pass filter, we can reconstruct the lung motion $\frac{\partial \sigma^t_L}{\partial t}$ by approximating the potentials in
Fig. 2. Configuration of current injection and voltage measurement and measured data separation with 8-channel EIT model. (a) the E-channel EIT model; (b) the conductivity $\sigma_L$ change in pink pixels of (a); (c) measured boundary data $V^{j,k}$; (d) decomposed data related to lung $V^{j,k}_L$; (e) the conductivity $\sigma_H$ change in green pixel of (a); (f) Fourier domain data $V^{j,k}$; (g) decomposed data related to heart and other effects $V^{j,k}_H$.

(11) by reference potentials and solving the resulting linear equation. We improve this strategy with the following monotonicity-based constraint. It is reasonable to assume that $\sigma_L^t$ is monotone with respect to $t$ in the sense that the lung conductivity is either increasing everywhere,

$$\frac{\partial}{\partial t} \sigma_L^t(x) \geq 0 \quad \text{for all } x \in \Omega, \quad (13)$$

or decreasing everywhere,

$$\frac{\partial}{\partial t} \sigma_L^t(x) \leq 0 \quad \text{for all } x \in \Omega. \quad (14)$$

For a symmetric matrix $A$ we write $A \geq 0$, if $A$ is semi-positive definite, that is, $a^TAa \geq 0$ for all vectors $a$.

From (11), we have

$$a^T \frac{d}{dt} V_L(t) a = - \int_\Omega \frac{\partial \sigma_L^t}{\partial t} \nabla \left( \sum_{j=1}^E a_j u_j^t \right) \cdot \nabla \left( \sum_{k=1}^E a_k u_k^t \right) dx$$

for all vectors $a = (a_1, \ldots, a_E)^T \in \mathbb{R}^E$.

We thus have the following monotonicity relations which are illustrated in Fig. 3.

$$\inf_{x \in \Omega} \frac{\partial \sigma_L^t}{\partial t}(x) \geq 0 \quad \Rightarrow \quad \frac{d}{dt} V_L \leq 0 \quad (16)$$

$$\sup_{x \in \Omega} \frac{\partial \sigma_L^t}{\partial t}(x) \leq 0 \quad \Rightarrow \quad \frac{d}{dt} V_L \geq 0. \quad (17)$$

The proposed method evaluates whether the lung is in the increasing or decreasing period and reconstructs the lung motion $\frac{\partial \sigma_L}{\partial t}$ using the additional global monotonicity-based constraint (13), resp., (14).

C. Imaging with global monotonicity-based constraints

Discretizing the imaging domain $\Omega$ into $\Omega = \bigcup_{p=1}^N T_p$ where $T_p$ is the $p$-th pixel, the conductivity distribution $\sigma_L^t$ in domain $\Omega$ can be expressed as $N \times 1$ vector $\boldsymbol{\sigma}_L^t$ (bold symbol). For a vector $\mathbf{b} = (b_1, \ldots, b_N)^T$, we write $\mathbf{b} \geq 0$, resp., $\mathbf{b} \leq 0$ if all components of the vector are non-negative, resp., non-positive. From the monotonicity assumption (13), resp., (14), the time differential $\frac{\partial \sigma_L^t}{\partial t}$ satisfies the following uniform positivity (or negativity) property:

$$\frac{\partial \sigma_L^t}{\partial t} \geq 0 \quad \text{or} \quad \frac{\partial \sigma_L^t}{\partial t} \leq 0. \quad (18)$$

This paper uses this monotonicity constraint (18) to solve the ill-conditioned linear system (called linearized EIT system) that arises from approximating the potentials in (11) by reference potentials:

$$\frac{d}{dt} V_L = \mathbb{S} \frac{\partial \sigma_L^t}{\partial t}, \quad (19)$$

where $\mathbb{S}$ is the sensitivity matrix ($E^2 \times N$ matrix) whose $(j-1) \times E + k, p)$ element is $S_{j,k}^{p} = -J_{r_p} \nabla u_0 \cdot \nabla u_0^p dx$, and $V_L(t)$
is the column concatenated data set of $\mathbb{V}_L(t)$ given by

$$\mathbb{V}_L = (V_{L,1}^{1,1}, \ldots, V_{L,E}^{1,1}, \ldots, V_{L,1}^{1,E}, \ldots, V_{L,E}^{1,E})^T.$$ \hfill (20)

The reference potential $u_0^j$ in the sensitivity matrix is chosen as the solution of (1)-(4) with the background conductivity distribution $\sigma_0$ which is herein chosen to be the homogeneous value $\sigma_0 := 1$.

We obtain the following globally monotonicity-based constrained reconstruction method:

- In the increasing period of conductivity ($\frac{d}{dt}\mathbb{V}_L \leq 0$) minimize \[ \|S \frac{\partial \sigma^t_L}{\partial t} - \frac{d}{dt}\mathbb{V}_L\| \text{ subject to } \frac{\partial \sigma^t_L}{\partial t} \geq 0. \] \hfill (21)

- In the decreasing period of conductivity ($\frac{d}{dt}\mathbb{V}_L \geq 0$) minimize \[ \|S \frac{\partial \sigma^t_L}{\partial t} - \frac{d}{dt}\mathbb{V}_L\| \text{ subject to } \frac{\partial \sigma^t_L}{\partial t} \leq 0. \] \hfill (22)

Since $\mathbb{V}_L$ can be expected to be positive definite, $\frac{d}{dt}\mathbb{V}_L \leq 0$ implies $\frac{d}{dt}\|\mathbb{V}_L\| \leq 0$, and $\frac{d}{dt}\mathbb{V}_L \geq 0$ implies $\frac{d}{dt}\|\mathbb{V}_L\| \geq 0$. In order to implement the minimization problems of (21) and (22), we discretize time $t$ as

$$t_1 < \cdots < t_{n-1} < t_{n+1} < \cdots$$

and compute the time difference $\sigma^t_L - \sigma^{t_{n-1}}_L$ by solving the following constraint minimization problem:

- In the case when $\|\mathbb{V}^{t_t}_L\| \leq \|\mathbb{V}^{t_{n-1}}_L\|$ (increasing conductivity period),

$$\begin{cases} \min \|S (\sigma^{t_t}_L - \sigma^{t_{n-1}}_L) - (\mathbb{V}^{t_t}_L - \mathbb{V}^{t_{n-1}}_L)\| \\
\text{subject to } \sigma^{t_t}_L - \sigma^{t_{n-1}}_L \geq 0. \end{cases} \hfill (23)$$

- In the case when $\|\mathbb{V}^{t_t}_L\| \geq \|\mathbb{V}^{t_{n-1}}_L\|$ (decreasing conductivity period),

$$\begin{cases} \min \|S (\sigma^{t_t}_L - \sigma^{t_{n-1}}_L) - (\mathbb{V}^{t_t}_L - \mathbb{V}^{t_{n-1}}_L)\| \\
\text{subject to } \sigma^{t_t}_L - \sigma^{t_{n-1}}_L \leq 0. \end{cases} \hfill (24)$$

D. Additional local monotonicity-based constraints

In the last two subsections, we used a data splitting approach and a monotonicity argument to obtain a global lower (upper) bound for the conductivity change in the increasing (decreasing) conductivity period. In this subsection, we use a refined monotonicity argument to obtain also an upper bound for the conductivity change for the increasing conductivity period (and, analogously, a lower bound for the decreasing conductivity period). The following approach is a new combination of the standard linearized reconstruction method with a sensitivity based variant of the monotonicity method developed in [22]. This is motivated by the recent paper [11] (see also [20]) which uses a sensitivity based variant of the factorization method (see [17], [21], [27]) to regularize the standard linearized reconstruction method.

For the $p$-th pixel $T_p \subset \Omega$, define the following $E \times E$ matrix

$$S^t_p = \begin{bmatrix} \int_{T_p} \nabla u_1^1 \cdot \nabla u_1^1 dx & \cdots & \int_{T_p} \nabla u_1^E \cdot \nabla u_1^E dx \\
\vdots & \ddots & \vdots \\
\int_{T_p} \nabla u_E^1 \cdot \nabla u_E^1 dx & \cdots & \int_{T_p} \nabla u_E^E \cdot \nabla u_E^E dx \end{bmatrix}$$

and note that, for $a = (a_1, \ldots, a_E)^T \in \mathbb{R}^E$,

$$a^T S^t_p a = \int_{T_p} \left| \sum_{j=1}^E a_j \nabla u_i^j \right|^2 dx. \hfill (25)$$

We will use the following quantitative version of the monotonicity relations (16) and (17). For $a = (a_1, \ldots, a_E)^T \in \mathbb{R}^E$,

$$\int_{\Omega} (\sigma^{t_{n-1}}_L - \sigma^{t_{n-1}}_p) \sum_{j=1}^E a_j \nabla u_i^j \left| dx \right. \geq a^T (\mathbb{V}_L(t_n) - \mathbb{V}_L(t_{n-1})) a$$

$$\geq \int_{\Omega} (\sigma^{t_{n-1}}_L - \sigma^{t_{n-1}}_p) \sum_{j=1}^E a_j \nabla u_i^{j-1} \left| dx \right., \hfill (26)$$

which is proven in the Appendix B.

Consider the increasing conductivity period, $\sigma^{t_{n-1}}_L \leq \sigma^{t_{n}}_L$.

If $\alpha > 0$ fulfills

$$-\alpha T_p \geq \sigma^{t_{n-1}}_L - \sigma^{t_{n}}_L,$$

then, by (25) and (26),

$$-\alpha S^t_p \geq \mathbb{V}_L(t_n) - \mathbb{V}_L(t_{n-1}).$$

Under the assumption that the conductivity change is constant on the pixel $T_p$ we hence obtain by contraposition that

$$-\alpha S^t_p \nabla L(t_n) - \mathbb{V}_L(t_{n-1}) \Rightarrow (\sigma^{t_{n-1}}_L - \sigma^{t_{n}}_L) |T_p| \leq \alpha.
$$

Hence, for each pixel we aim to find a smallest possible $\alpha \geq 0$ with $-\alpha S^t_p \nabla L(t_n) - \mathbb{V}_L(t_{n-1})$. Collecting the values for each pixel in a vector $\alpha$, we arrive at the following constrained reconstruction method:

- In the case when $\|\mathbb{V}^{t_t}_L\| \leq \|\mathbb{V}^{t_{n-1}}_L\|$ (increasing conductivity period),

$$\begin{cases} \min \|S (\sigma^{t_t}_L - \sigma^{t_{n-1}}) - (\mathbb{V}^{t_t}_L - \mathbb{V}^{t_{n-1}}_L)\| \\
\text{subject to } \alpha \geq \sigma^{t_t}_L - \sigma^{t_{n-1}}_L \geq 0. \end{cases} \hfill (27)$$

Analogously, we obtain for the decreasing conductivity period, $\sigma^{t_t}_L \leq \sigma^{t_{n-1}}_L$, that, for all $\beta \geq 0$,\n
$$\beta S^t_p \nabla L(t_n) - \mathbb{V}_L(t_{n-1}) \Rightarrow (\sigma^{t_{n-1}}_L - \sigma^{t_{n}}_L) |T_p| \leq \beta,$$

so that we aim to find a smallest possible $\beta \geq 0$ with $\beta S^t_p \nabla L(t_n) - \mathbb{V}_L(t_{n-1})$ and collect the values for each pixel in a vector $\beta$.

- In the case when $\|\mathbb{V}^{t_t}_L\| \geq \|\mathbb{V}^{t_{n-1}}_L\|$ (decreasing conductivity period),

$$\begin{cases} \min \|S (\sigma^{t_t}_L - \sigma^{t_{n-1}}) - (\mathbb{V}^{t_t}_L - \mathbb{V}^{t_{n-1}}_L)\| \\
\text{subject to } -\beta \leq \sigma^{t_t}_L - \sigma^{t_{n-1}}_L \leq 0. \end{cases} \hfill (28)$$

For the implementation of these additional local monotonicity-based constraints, we approximate the potentials $u_i^j$ in the definition of $S^t_p$ by reference potentials $u_0^j$ as this is done in the above-described linearized reconstruction methods. Accordingly $S^t_p$ and $S^{t_{n-1}}_p$ are approximated by the matrix $S_p$, which is obtained by a rearrangement of the $p$-th column of the sensitivity matrix $S$ defined in the previous subsection. The smallest possible values for $\alpha$, resp., $\beta$ on each pixel are determined by a binary search method.
III. NUMERICAL SIMULATIONS

To validate the analysis in the previous section, we perform several numerical experiments. For the forward simulations we used the point electrode model, cf. [18].

A. Numerical algorithm

The constraint minimization problem with the non-negative constraint (23) and (24) can be viewed as non-negative least square (NNLS) problem [8], [9], [37]. Numerous experiments verified that enforcing a non-negative constraint could lead to more accurate approximate solution [16], [31]. We adopt the following algorithm to solve problems (23) and (24)

- Initialization: Assume \( \sigma_L^{t_0} = \sigma_0 \) at time \( t = t_0 \), and reference data is \( \nabla L(t_0) \). Compute sensitivity matrix \( S \).
- For time \( t = t_n, n = 1,2, \ldots \)
  1. Read measured boundary voltage data \( V(t_n) \).
  2. Apply low pass filter to get separated data \( \nabla L(t_n) \) and the time difference data \( \nabla L(t_n) - \nabla L(t_{n-1}) \).
  3. Evaluate the values of matrix norm \( ||\nabla L(t_n)|| \) and \( ||\nabla L(t_{n-1})|| \).
    - (a) If \( ||\nabla L(t_n)|| \leq ||\nabla L(t_{n-1})|| \), apply non-negative least square algorithm [8] to (23).
    - (b) If \( ||\nabla L(t_n)|| > ||\nabla L(t_{n-1})|| \), apply non-negative least square algorithm to (24).
  4. Output: For certain given \( n \geq 1, \sigma_L^{t_n} \) solve problems (23) and (24).

Stop when \( n \) reaches the set time step.

To solve (27) and (28), we used similar procedure as above but a little bit different in step 3. (a) and (b). The step 3. (a) and (b) should be adapted as:

- (a) If \( ||\nabla L(t_n)|| \leq ||\nabla L(t_{n-1})|| \), compute \( \alpha \) and apply linear least square algorithm [12] to (27).
- (b) If \( ||\nabla L(t_n)|| > ||\nabla L(t_{n-1})|| \), compute \( \beta \) and apply linear least square algorithm to (28).

With adding constraints of the conductivity change, solution of (23), (24) and (27), (28) will provide more accurate image compare to conventional method. Numerical and phantom experiments results will be shown in next sections.

B. 2D numerical example

We consider the 16-channel EIT system for the chest model as shown in Fig. 4 where the domain \( \Omega \) is in the region of rectangle \([0,7.5] \times [0,5]\). The electrodes are indexed by \( \mathcal{E}_i, i = 1, \ldots, 16 \), and the voltage measurements \( V^k, k = 1, \ldots, 16 \) are taken between \( \mathcal{E}_k \) and \( \mathcal{E}_{k+1} \) for each injection. In this simple model, we add three anomalies, one heart modeled as circular anomaly \( H = \{(x, y)|(x-3.7)^2 + (y-1.7)^2 \leq 0.7^2\} \) and two lungs modeled as ellipse anomalies \( L_1 = \{(x, y)|(x-5.6)^2 + (y-0.5)^2 \leq 1\} \) and \( L_2 = \{(x, y)|(x-5.6)^2 + (y-3)^2 \leq 1\} \). We tested the case that the conductivity in lung and heart region is changing with time. The basic settings for conductivity variation in this case is displayed in the TABLE I. Fig. 5 (a) shows the distribution of measured boundary data and Fig. 5 (b) shows difference boundary data with time for the 6-th current injection.

We are going to separate the boundary time dependent data into two parts, one part is related to the lung ventilation signal and the other part is related to the cardiac motion and other effects signals. Since the pulmonary activity ratio and heart ratio are different, the low pass filtered boundary data will mainly related to pulmonary activity. Corresponding to boundary data in Fig. 5 (b), we display the separated data in the following Fig. 6.

To display the numerical simulation results, we select 10 frames of reconstruction for time be vector \( t = (t_1, t_2, \ldots, t_{10}) = (0, 0.1, 0.3, \ldots, 1.9) \) in one period of lung ventilation. The true distribution of conductivity are shown in the first two rows of Fig. 7. The reconstruction from standard algorithm (19) use original boundary data \( V \) are displayed in the third and fourth rows of Fig. 7.
Since we are interested in lung imaging of ventilation, only the low frequency part of data and reconstruction will be considered in the follows. The reconstruction results from standard algorithm (19) use separated boundary data \( V_L \), are displayed in the fifth and sixth rows of Fig. 7. The reconstruction from proposed global monotonicity-based method (23) and (24) are displayed in the seventh and eighth rows of Fig. 7.

For the proposed local monotonicity-based method (27) and (28), we first showed the estimation of \( \alpha \) and \( \beta \) in the ninth and tenth rows of Fig. 7. With the extra information of \( \alpha \) and \( \beta \), we show the reconstruction using (27) and (28) in the eleventh and twelfth rows of Fig. 7.

![32-electrodes configuration for 3D simulation.](image)

**Fig. 8.** 32-electrodes configuration for 3D simulation.

We can see that the reconstructed images in Fig. 7 from standard method have serious boundary artifacts either with data \( V \) or separated data \( V_L \). On the other hand, the proposed global and local monotonicity-based methods produce much improved results for the reconstruction of lungs without boundary artifacts. Compare the two proposed methods, global and local monotonicity based methods, local monotonicity-based method give more accurate position and size estimates of lung. It shows clearly that \( \alpha \) and \( \beta \) provide upper bound and lower bound of conductivity change in increase and decrease period, respectively. This extra information of conductivity change make it possible to enhance the image quality.

### C. 3D numerical example

We did 3D numerical simulation using our proposed methods. In order to compare with phantom experiments, we used the 32-channel numerical model which is displayed in Fig. 8. The numerical model size is 26cm in length, 17cm in width and 12cm in height. 32 electrodes are put at two layers (height are at \( z = 3cm \) and \( z = 9cm \)) with each layer 16 electrodes. Currents are injected adjacenty and measured in the same way. We put two cylinders in the domain to simulate the two lungs. The center of two cylinders are at positions \((-8, -2, 5.5)cm\) and \((8, -2, 5.5)cm\), respectively. And the size of two cylinders are same at the same time and shown in TABLE II. The conductivity value of these two cylinders are set to be 0.5 and background conductivity is 1.

![32-electrodes configuration for 3D simulation.](image)

**Fig. 7.** Reconstruction of conductivity change in 2D: the 1st row is true distribution of conductivity change; the 2nd row is the reconstruction using standard algorithm (19) with data \( V \); the 3rd row is the reconstruction using standard algorithm (19) with separated data \( V_L \); the 4th row is the reconstruction from proposed global monotonicity-based method (23) and (24); the 5th row is the evaluation of \( \alpha \) and \( \beta \); the 6th row is the reconstruction from proposed local monotonicity-based method (27) and (28); the last row is the index of time.

The reconstruction results are shown in the Fig. 9 and Fig. 10 with xy-slice images at \( z = 9cm \) and \( z = 3cm \), respectively. We have the true distribution of conductivity change in the first row of Fig. 9 and Fig. 10 and corresponding reconstructions using standard method are put in the second row of Fig. 9 and Fig. 10.

Based on the proposed global monotonicity-based method (23) and (24), we numerically show the results in the third row of Fig. 9 and Fig. 10.

In order to do reconstruction using the proposed local monotonicity-based method (27) and (28), we first estimated \( \alpha \) and \( \beta \). With the additional information of \( \alpha \) and \( \beta \), the reconstruction of local monotonicity-based method are displayed in the fourth row of Fig. 9 and Fig. 10.

**IV. PHANTOM EXPERIMENTS**

Experiments were conducted with chest shape phantom using 32-channel Swisstom pioneer EIT system shown in Fig.1. In order to simulate the low conductivity of lung and monotonically change of lung size, three size of well trimmed radish were used shown in Fig.11. We inject current adjacenty with 5mA current at frequency 50kHz and the boundary voltage data are measured 10 frames per second.

Specifically, our phantom size is 26cm in length, 17cm in width and 12cm in height. We fill the phantom with salt water with conductivity 0.32s/m. And we make the radish in large, medium and small size so that we can simulate the size change...
during lung ventilation. The detail information of these size of radish is shown in Table III.

The Swisstom EIT system provides a full set of measurements, \( V_{j,k}(t) \), \( j, k = 1, \ldots, 32 \) for each time frame. However, the voltage data on current-driven electrodes is known to be specifically prone to errors and affected by unknown contact impedances, so that usually only \( 32 \times 29 \) measurements \( (V_{j,k}(t) \text{ with } |j - k| > 1) \) are used for the reconstruction. In our algorithms we used the reduced set of \( 32 \times 29 \) measurements for evaluating the residuum norm

\[
\| \mathcal{S}(\sigma_{L}^{n} - \sigma_{L}^{n-1}) - (V_{L}^{n} - V_{L}^{n-1}) \|,
\]

but the full \( 32 \times 32 \) measurement matrix for calculating \( \| V_{L}^{n} \| \),

We can see that even the results from the proposed methods are not perfect, they are relatively good compared to the results from standard method considering the shape and position of the anomaly. The experiment results clearly show that the proposed monotonicity-based methods can catch the monotonicity change of conductivity even in real experiment.

V. CONCLUSIONS

The proposed reconstruction method allows lung EIT to visualize the monotonically varying conductivity distribution during inhalation and exhalation. It is based on the assumption that lung conductivity monotonically decreases during inhalation (due to the air flowing into the lungs) and monotonically increases during exhalation (due to the air leaving the

<table>
<thead>
<tr>
<th>Time</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>( t_3 )</th>
<th>( t_4 )</th>
<th>( t_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>9.5cm</td>
<td>10cm</td>
<td>11cm</td>
<td>10cm</td>
<td>9.5cm</td>
</tr>
<tr>
<td>Radius</td>
<td>1.5cm</td>
<td>2.2cm</td>
<td>3.5cm</td>
<td>2.2cm</td>
<td>1.5cm</td>
</tr>
</tbody>
</table>

**Table III**

SIZE OF RADISH FOR EXPERIMENTS AT DIFFERENT TIME.
lungs). The periodicity of lung ventilation is used to extract its associated current-voltage data; all voltage differences between electrodes increase during inhalation and decrease during exhalation regardless of the injection currents. This correlation between the time-differential of the current-voltage map and the changes of conductivity can be enforced in the reconstruction algorithm as a constraint.

We know that the inverse problem of EIT is ill-posed, and therefore any least square method by data-fitting alone may not be able to provide useful images. This means that the boundary current-voltage data alone are insufficient to achieve robust reconstructions for making clinically useful images. Due to the inherent methodological limitation, the EIT reconstruction algorithm requires a strategy that balances data fitting and a suitable regularization by imposing a certain constraint on the expected image. The proposed method uses monotonicity as such a regularization.

Although EIT has limited resolution, its unique advantage lies in its capability for continuous monitoring at the bedside. Given the drawbacks of EIT (e.g., technical difficulties of the ill-posedness related to EIT data being insufficient to probe local conductivity changes), we need to focus on a robust reconstruction method to allow this technique to provide indispensable information in clinical medicine.

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APPENDIX

A. Proof of the identities (7) and (8).
First note that

\[ V^{j,k}(t) = u^j_t|_{x_k} - u^j_t|_{x_{k+1}} = \int_{\partial \Omega} u^j_t(\sigma^t \nabla u^j_t \cdot \mathbf{n}) ds = \int_{\Omega} \sigma^t \nabla u^j_t \cdot \nabla u^k_t dx, \]

which shows (7) and \( V^{j,k}(t) = V^{k,j}(t) \). With the same argument we obtain that

\[ V^{j,k}(t + \delta t) = u^j_t|_{x_k} - u^j_t|_{x_{k+1}} = \int_{\partial \Omega} u^j_t(\sigma^{t+\delta t} \nabla u^j_t \cdot \mathbf{n}) ds = \int_{\Omega} \sigma^{t+\delta t} \nabla u^j_t \cdot \nabla u^k_t dx, \]

and

\[ V^{j,k}(t + \delta t) = V^{k,j}(t + \delta t) = \int_{\Omega} \sigma^{t} \nabla u^j_t \cdot \nabla u^k_{t+\delta t} dx. \]

Hence,

\[ \frac{d}{dt} V^{j,k}(t) = - \int_{\Omega} \frac{\partial \sigma^t}{\partial t} \nabla u^j_t \cdot \nabla u^k_t dx, \]

and it follows that

\[ \frac{d}{dt} V^{j,k}(t) = - \int_{\Omega} \frac{\partial \sigma^t}{\partial t} \nabla u^j_t \cdot \nabla u^k_t dx, \]

which is the asserted identity (8). \( \Box \)

B. Proof of estimate (26).

This type of monotonicity estimate goes back to [25], [26], see also [19], [22], [23] for recent applications. Let \( a_1, \ldots, a_E \in \mathbb{R} \). For brevity, we write \( u^n_a := \sum_{j=1}^E a_j u^j_t \). As in the proof of (7) and (8) we have that

\[ a^T \nabla_L (t_{n-1}) a = \int_{\Omega} \sigma^{t_{n-1}} \nabla u^n_{t_{n-1}} \cdot \nabla u^n_{t_{n-1}} dx, \]

\[ a^T \nabla_L (t_n) a = \int_{\Omega} \sigma^{t_n} \nabla u^n_{t_n} \cdot \nabla u^n_{t_n} dx = \int_{\Omega} \sigma^{t_{n-1}} \nabla u^n_{t_{n-1}} \cdot \nabla u^n_{t_{n-1}} dx. \]

From

\[ 0 \leq \int_{\Omega} \sigma^{t_{n-1}} |\nabla u^n_{t_{n-1}} - \nabla u^n_{t_n}|^2 = \int_{\Omega} \sigma^{t_{n-1}} |\nabla u^n_{t_{n-1}}|^2 - 2 \int_{\Omega} \sigma^{t_{n-1}} \nabla u^n_{t_{n-1}} \cdot \nabla u^n_{t_n} + \int_{\Omega} \sigma^{t_{n-1}} |\nabla u^n_{t_n}|^2. \]

\[ = a^T (\nabla_L (t_{n-1}) - \nabla_L (t_n)) a + \int_{\Omega} (\sigma^{t_{n-1}} - \sigma^{t_n}) |\nabla u^n_{t_n}|^2; \]

it follows that

\[ a^T (\nabla_L (t_n) - \nabla_L (t_{n-1})) a \leq \int_{\Omega} (\sigma^{t_{n-1}} - \sigma^{t_n}) |\nabla u^n_{t_n}|^2, \]

which is the first inequality in (26). The second inequality in (26) follows from interchanging \( t_n \) and \( t_{n-1} \).
REFERENCES


