Hausdorff Dimension of Rings

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HAUSDORFF DIMENSION OF RINGS

Dietmar Kahnert

Dedicated to Professor Dr. Bodo Volkmann on his 85. birthday

Abstract

In context with the problem of Volkmann whether a subfield $K$ of $\mathbb{R}$ exists with Hausdorff dimension $\dim K \in (0, 1)$, Falconer has proven that there is no subring $S$ with $1/2 < \dim S < 1$ which is an analytic set. We prove that $S = \mathbb{R}$ for every such subring $S$ with $\dim S > 0$.

1 A problem of Volkmann

A function $h : [0, \infty) \to [0, \infty]$ is called Hausdorff function if the following is valid: $h(0) = 0$, $h(t) > 0$ if $t > 0$, $h(a) \leq h(b)$ if $a \leq b$ and $\lim_{t \to 0} h(t) = 0$.

Let $H$ be the set of all Hausdorff functions. For each $h \in H$ and every subset $A$ of $\mathbb{R}^n$ is the outer measure

$$L_h(A) = \lim_{q \to 0} \inf \left\{ \sum_{i=1}^{\infty} h(d(A_i)) : A = \bigcup_{i=1}^{\infty} A_i, d(A_i) \leq q \text{ for all } i \in \mathbb{N} \right\}$$

defined. Let $d(A_i)$ be the diameter of $A_i$. Souslin sets (also called “analytic sets”), especially Borel sets, are $L_h$-measurable. If $h(t) = t^\alpha$ ($\alpha > 0$), $L_\alpha = L_h$ is the $\alpha$-dimensional Hausdorff measure and

$$\dim A = \sup\{\alpha : L_\alpha(A) > 0\}$$

the Hausdorff dimension of $A$.

The field problem of Volkmann [18]

Is there a subfield $K$ of the field $\mathbb{R}$ of real numbers with $0 < \dim K < 1$?

The problem still remains open.

Result of Falconer ([5] and [7])

No Souslin subring $S$ of $\mathbb{R}$ exists with $1/2 < \dim S < 1$.

The result of Falconer emerges from his theorems regarding the projections of subsets $E$ of $\mathbb{R}^2$ onto $\mathbb{R}$ and over distance sets $D(E) = \{ |x - y| : x, y \in E \}$. They were won with the help of Fourier transformations. The following generalization should be treated here (Theorem 2):

For each Souslin subring $S$ of $\mathbb{R}$ with $\dim S > 0$ is $S = \mathbb{R}$.
2 Special subfields of $\mathbb{R}$

2.1 Small subfields

An uncountable $F_\sigma$-subfield $K$ of $\mathbb{R}$ with $L_1(A) = 0$ is constructed in a paper of Souslin [17]. In measure-theoretical view one can win small subfields $K$ of $\mathbb{R}$ with help of the metric dimension of Wegmann [19]. Wegmann defines for subsets $A$ of $\mathbb{R}^n$ and $q > 0$

$$N(A, q) = \min \{ k : \text{There are sets } A_1, \ldots, A_k \text{ such that } \bigcup_{i=1}^k A_i = A; d(A_i) \leq q \text{ if } 1 \leq i \leq k \}$$

and

$$m\text{-dim } A = \sup \{ s : \text{If } \bigcup_{i=1}^\infty A_i = A, \text{ then exists } i \in \mathbb{N} \text{ with } \limsup_{q \to 0} N(A_i, q)^s > 0 \}.$$ 

In the book of Mattila [14] $m\text{-dim} = \dim_p$ is called upper packing dimension. Clearly $\dim \leq m\text{-dim}$.

If $A$ is a subset of $\mathbb{R}$ and $K(A)$ the smallest subfield of $\mathbb{R}$ containing $A$:

$m\text{-dim } A = 0 \to m\text{-dim } K(A) = 0$ (Kahneret [9]).

With the method used in [9] we can prove: If $g, h \in H$ and $\lim_{q \to 0} h(t)/g(t)^n = 0$ for all $n \in \mathbb{N}$, then $\lim_{q \to 0} N(A, q)g(q) = 0 \to L_h(K(A)) = 0$ for every compact subset $A$ of $\mathbb{R}$.

An uncountable subset $A$ of $\mathbb{R}$ is called Lusin set, if every uncountable subset of $A$ is of second category. For Lusin sets $A$ is $L_h(A) = 0$ for each $h \in H$.

With the help of the continuum hypothesis it is possible to get subfields of $\mathbb{R}$ which are Lusin sets (Erdős [4]).

2.2 Big subfields

According to an idea of Zygmund, in the paper of Souslin [17], the existence of a non-$L_1$-measureable subfield $K$ of $\mathbb{R}(L_1(K) > 0, \dim K = 1)$ can be proven with the help of the axiom of choice.

An uncountable subset $A$ of $\mathbb{R}$ is called Sierpinski set (dual to Lusin set), if every uncountable subset of $A$ is of positive outer $L_1$-measure. With the help of the continuum hypothesis Erdős and Volkmann [3] proved the existence of fields which are Sierpinski sets.

In the latter mentioned paper Erdős und Volkmann constructed for each $\alpha \in (0, 1)$ additive $F_\sigma$-subgroups $G(\alpha)$ of $\mathbb{R}$ with $\dim G(\alpha) = \alpha (= m\text{-dim } G(\alpha))$. This result led to the supposition that a corresponding statement could be true for subfields of $\mathbb{R}$.

3 Subfields $K$ of the complex numbers $\mathbb{C}$ of finite degree

A field $E$ containing a field $F$ can be regarded as an $F$-vector space. We write $E : F$ for the dimension. We refer in the following to the book of Hornfeck [8].

3.1 $\mathbb{C}$ is normal over $K$ if $\mathbb{C} : K < \infty$

Let $G(\mathbb{C} : K)$ be the group of automorphism $\varphi$ of $\mathbb{C}$ with $\varphi(x) = x$ for all $x \in K$. The field $\mathbb{C}$ is called normal over $K$ if $K$ is the fixed field of $G(\mathbb{C} : K)$ (other authors name in this case $\mathbb{C}$
Galois over $K$). There is $\alpha \in C$ with $C = K(\alpha)$ (Theorem 3a, 61.2). The minimal polynomial $f(x) \in K[x]$ of $\alpha$ splits in $C[x]$:

$$f(x) = (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_n), \quad (\alpha_1 = \alpha).$$

Therefore is $K(\alpha_1, \alpha_2, \ldots, \alpha_n) = K(\alpha) = C$ a splitting field of $f(x)$ (Lemma in 58.1). $C$ is normal over $K$ (Theorem 7, 65.2) and $|G(C : K)| = C : K$.

The field $C$ has two continuous automorphisms $\varphi_1$ and $\varphi_2$ with $\varphi_1(x) = x$ and $\varphi_2(x) = x$ for all $x \in C$. The field $\mathbb{R}$ has only one automorphism.

### 3.2 Continuous additive functions

We shall prove Theorem 1 using the following special-case of a result of Ostrowski [15] and give the proof due to Kestelman [11].

**Ostrowski’s theorem** If $f : \mathbb{R}^n \to \mathbb{R}^n$ is additive ($f(x + y) = f(x) + f(y)$) and bounded on a $L_n$-measurable set $A$ with $L_n(A) > 0$, then $f$ is continuous.

**Proof:** Let $M = \sup \{|f(x)| : x \in A\}$. After a known result of Steinhaus, the set $A - A$ contains a ball around the origin with radius $r$. Every $x \in \mathbb{R}^n$ with $|x| < r$ can be written as $x = a - b$ $(a, b \in A)$ and therefore

$$|f(x)| = |f(a) - f(b)| \leq 2M.$$

For $n \in \mathbb{N}$ and $|x| < r/n$ is $|nx| < r$, $|f(nx)| = n|f(x)| \leq 2M$ and $|f(x)| \leq 2M/n$. Therefore $f$ is continuous in the origin and consequently everywhere continuous. □

### 3.3 Souslin subfields $K$ of $C$ with $C : K < \infty$

The properties of analytic sets mentioned here are treated for example in the book of Parthasarathy [16].

**Theorem 1** Souslin subfields $K$ of $C$ with $C : K < \infty$ are only $K = \mathbb{R}$ and $K = C$.

**Proof:** Let $b_1, \ldots, b_n$ be a basis of $C$ over $K$:

$$C = b_1K + b_2K + \ldots + b_nK.$$ 

For $r > 0$ we define

$$A(r) = \{b_1z_1 + b_2z_2 + \ldots + b_nz_n : (z_1, \ldots, z_n) \in K^n, |z_1| + \ldots + |z_n| \leq r\},$$

$$B(r) = \{(z_1, z_2, \ldots, z_n) \in C^n : |z_1| + \ldots + |z_n| \leq r\} \text{ and}$$

$$C(r) = B(r) \cap K^n.$$

Then $B(r)$ is compact, $K^n$ and $C(r)$ are Souslin sets in $C^n$. We show: $A(r)$ is analytic.

For $z = (z_1, \ldots, z_n)$ let

$$f(z) = b_1z_1 + \ldots + b_nz_n \quad (z \in C^n) \text{ and}$$

$$g(z) = f(z) \quad (z \in C(r)).$$
Since $f$ is continuous it follows that for every Borel set $D$ in $\mathbb{C}$
\[ f^{-1}(D) \] is a Borel set in $\mathbb{C}^n$ and
\[ g^{-1}(D) = f^1(D) \cap C(r) \] is a Borel set in $C(r)$.

Therefore $g$ is Borel measurable and $A(r) = g(C(r))$ analytic.

For $\varphi \in G(\mathbb{C} : K)$ (C is normal over $K$) and $M = \max \{|b|, \ldots, b|\}$ is
\[ |\varphi(z)| \leq Mr \text{ if } z \in A(r). \]

There is $r$ with $L_2(A(r)) > 0$. By Ostrowski's result $\varphi$ is continuous, therefore $G(\mathbb{C} : K) = \{\varphi_1\}$ and $K = \mathbb{C}$, or $G(\mathbb{C} : K) = \{\varphi_1, \varphi_2\}$ and $K = \mathbb{R}$. □

The analytic property of $K$ is not required with the following result.

**Artin's Theorem [1]** For every subfield $K$ of $\mathbb{C}$ with $1 < \mathbb{C} : K < \infty$ is $\mathbb{C} : K = 2$. (Especially there exists no subfield $K$ of $\mathbb{R}$ with $1 < \mathbb{R} : K < \infty$.)

The following special-case, that can be used in the Section 5, can easily be proven. There is no subfield $K$ of $\mathbb{R}$ with $\mathbb{R} : K = 2$.

Suppose that $K$ is a subfield of $\mathbb{R}$ and $\mathbb{R} : K = 2$. Then there is a real number $\alpha$ with $\mathbb{R} = K(\alpha)$. Let $f(x)$ be the minimal polynomial of $\alpha$ and
\[ f(x) = (x - \alpha_1)(x - \alpha_2), \quad (\alpha_1 = \alpha). \]

Then $\alpha_2$ must be a real number.

Thus is $K(\alpha_1, \alpha_2) = K(\alpha) = \mathbb{R}$, $K(\alpha_1, \alpha_2)$ a splitting field, $\mathbb{R}$ normal over $K$ and $|G(\mathbb{R} : K)| = 2$.

But $\mathbb{R}$ has only one automorphism.

## 4 The Main Result

Let $A$ be a non-empty subset of $\mathbb{R}$ and $\mathbb{R}(A)$ the subring of $\mathbb{R}$ generated by $A$.

**Theorem 2** For every closed subset $A$ of $\mathbb{R}$ with $\dim A > 0$ is $R(A) = \mathbb{R}$.

By results of Besicovitch and Davies [2] any Souslin subset $A$ of $\mathbb{R}^n$ with $L_\alpha(A) > 0$ contains a closed subset $B$ with $0 < L_\alpha(B) < \infty$. For every Souslin subring $S$ of $\mathbb{R}$ with $\dim S > 0$ is therefore $S = \mathbb{R}$.

We use the following theorems of Marstrand to prove $\mathbb{R} : K(A) < \infty$ for every set $A$ of Theorem 2.

For subsets $E$ of $\mathbb{R}^2$ and $t \in \mathbb{R}$ be
\[ E(t) = \{x + ty : (x, y) \in E\}. \]

**Projection theorem (Marstrand [12])** Let $E$ be a Souslin subset of $\mathbb{R}^2$ with $\dim E = \alpha$:

- a) $\alpha \leq 1 : \dim E(t) = \alpha$ for almost all $t \in \mathbb{R}$,
- b) $\alpha > 1 : L_1(E(t)) > 0$ for almost all $t \in \mathbb{R}$.

A potential theoretic proof was given by Kaufmann [10]. Generalizations can be found in the books of Falconer [6] and Mattila [14].
**Product theorem (Marstrand [13])** For any subsets $A$ and $B$ of $\mathbb{R}^n$

$$\dim A \times B \geq \dim A + \dim B.$$  

A generalization of the product formula for general metric spaces was proven by Wegmann [19].

**Proof of Theorem 2.**

1. With possibly multiple applications of the theorems of Marstrand one proves the following assertion:

There are real numbers $b_1, \ldots, b_n$ with $L_1(b_1A + \ldots + b_nA) > 0$.

Let $b_1 = 1$ and $A_1 = A$. In the case $L_1(A) > 0$ there is nothing to prove. May $A_k = b_1A + \ldots + b_kA$ be defined and $L_1(A_1) = \ldots = L_1(A_k) = 0$.

In the case $\dim A_k \times A > 1$ exists by the projection theorem a real number $b_{k+1}$ with $L_1(A_k + b_{k+1}A) > 0$ and the assertion is verified.

In the case $\dim A_k \times A \leq 1$ there exists by the projection theorem a real number $b_{k+1}$ with $\dim A_k + b_{k+1}A = \dim A_k \times A$. For $A_{k+1} = A_k + b_{k+1}A$ is by the product formula

$$\dim A_{k+1} \geq \dim A_k + \dim A \geq (k + 1) \dim A.$$  

After finite steps, one arrives at the assertion.

2. If $U$ is the additive subgroup of $\mathbb{R}$ generated by $A$, then

$$G = b_1U + \ldots + b_nU$$

is a group, by the theorem of Steinhaus a neighborhood of 0 and therefore $G = \mathbb{R}$. For $S = R(A)$ and for the $F_\sigma$-field

$$K = \{s/t : s, t \in S, t \neq 0\} = K(A)$$

is

$$b_1K + \ldots + b_nK = \mathbb{R}, \mathbb{R} : K \leq n$$

and therefore $K = \mathbb{R}$ (Artin’s theorem, Theorem 1).

Let $b_1 = s_1/t_1, \ldots, b_n = s_n/t_n(s_i, t_i \in S; t_1t_2\ldots t_n \neq 0)$.

Multiplying $b_1S + \ldots + b_nS = \mathbb{R}$ with $t_1t_2\ldots t$ we get

$$\mathbb{R} = d_1S + \ldots + d_nS = S(d_1, \ldots, d_n \in S), \mathbb{R} = S.$$

\[\square\]

5 **A special case of Theorem 2**

For every closed subset $A$ of $\mathbb{R}$ with $\dim A > 1/2$ is $R(A) = \mathbb{R}$.

**Proof:** It is $\dim A \times A \geq 2 \dim A > 1$. By the projection theorem (part b) there exists a real number $t$ with

$$L_1(A + tA) > 0.$$  

Let $S$ be the $F_\sigma$-ring $R(A)$. Then $L_1(S + tS) > 0$.  

5
By the theorem of Steinhaus is the additive group
\[ S + tS = (S + tS) - (S + tS) \]
neighborhood of 0 and therefore \( S + tS = \mathbb{R} \).
For the field \( K = K(A) = \{a/b : a, b \in S, b \neq 0\} \) is
\[ K + tK = \mathbb{R}, \quad \mathbb{R} : K \leq 2 \]
and therefore \( K = \mathbb{R} \) (\( \mathbb{R} : K = 2 \) is not possible).
Be \( t = a/b(a, b \in S; b \neq 0) \). Then \( bS + aS = \mathbb{R} = S \). \( \square \)

6 Problems

- Is there a Souslin subfield \( K \) of \( \mathbb{C} \) with
  \[ 0 < \dim K < 2 \text{ and } \dim K \neq 1? \]
- Is there a subfield \( K \) of \( \mathbb{R} \) with \( 0 < m \cdot \dim K < 1? \)
- Which possibilities are there for a subfield \( K \neq \mathbb{R} \) of \( \mathbb{C} \) with \( \mathbb{C} : K = 2 \) concerning \( \dim K, m \cdot \dim K \) and the Baire category of \( K \)?

References


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